9.2 Z TEST OF HYPOTHESIS FOR THE MEAN (σ KNOWN)

Now that you have been introduced to hypothesis-testing methodology, recall that in the “Using Statistics” scenario on page 300, the Oxford Cereal Company wants to determine whether the cereal-filling process is working properly (that is, whether the mean fill throughout the entire packaging process remains at the specified 368 grams and no corrective action is needed). To evaluate the 368-gram requirement, you take a random sample of 25 boxes, weigh each box, and then evaluate the difference between the sample statistic and the hypothesized population parameter by comparing the mean weight (in grams) from the sample to the expected mean of 368 grams specified by the company. For this filling process, the null and alternative hypotheses are

\[ H_0: \mu = 368 \]
\[ H_1: \mu \neq 368 \]

When the standard deviation \( \sigma \) is known, you use the Z test if the population is normally distributed. If the population is not normally distributed, you can still use the Z test if the sample size is large enough for the Central Limit Theorem to take effect (see section 7.2). Equation (9.1) defines the Z-test statistic for determining the difference between the sample mean \( \bar{X} \) and the population mean \( \mu \) when the standard deviation \( \sigma \) is known.

**Z TEST OF HYPOTHESIS FOR THE MEAN (σ KNOWN)**

\[ Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \]  \hspace{1cm} (9.1)

In Equation (9.1) the numerator measures how far (in an absolute sense) the observed sample mean \( \bar{X} \) is from the hypothesized mean \( \mu \). The denominator is the standard error of the mean, so \( Z \) represents the difference between \( \bar{X} \) and \( \mu \) in standard error units.

The Critical Value Approach to Hypothesis Testing

The observed value of the Z test statistic, Equation (9.1), is compared to critical values. These critical values are expressed as standardized Z values (i.e., in standard-deviation units). For example, if you use a level of significance of 0.05, the size of the rejection region is 0.05. Because the rejection region is divided into the two tails of the distribution (this is called a two-tail test), you divide the 0.05 into two equal parts of 0.025 each. A rejection region of 0.025 in each tail of the normal distribution results in a cumulative area of 0.025 below the lower critical value and a cumulative area of 0.975 below the upper critical value. According to the cumulative standardized normal distribution table (Table E.2), the critical values that divide the rejection and nonrejection regions are −1.96 and +1.96. Figure 9.2 illustrates that if the mean is
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actually 368 grams, as \( H_0 \) claims, then the values of the test statistic \( Z \) have a standardized normal distribution centered at \( Z = 0 \) (which corresponds to an \( \bar{X} \) value of 368 grams). Values of \( Z \) greater than +1.96 or less than −1.96 indicate that \( \bar{X} \) is so far from the hypothesized \( \mu = 368 \) that it is unlikely that such a value would occur if \( H_0 \) were true.

**FIGURE 9.2**
Testing a Hypothesis about the Mean (σ Known) at the 0.05 Level of Significance

Therefore, the decision rule is

Reject \( H_0 \) if \( Z > +1.96 \)

or if \( Z < -1.96; \)
otherwise do not reject \( H_0 \).

Suppose that the sample of 25 cereal boxes indicates a sample mean \( \bar{X} = 372.5 \) grams and the population standard deviation \( \sigma \) is assumed to be 15 grams. Using Equation (9.1) on page 306:

\[
Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{372.5 - 368}{\frac{15}{\sqrt{25}}} = +1.50
\]

Because the test statistic \( Z = +1.50 \) is between −1.96 and +1.96, you do not reject \( H_0 \) (see Figure 9.3). You continue to believe that the mean fill amount is 368 grams. To take into account the possibility of a Type II error, you state the conclusion as “there is insufficient evidence that the mean fill is different from 368 grams.”

**FIGURE 9.3**
Testing a Hypothesis about the Mean (σ Known) at 0.05 Level of Significance